Grade 5 Mathematics Standards

Critical Areas for COHERENCE in Mathematics in Grade 5

In Grade 5, instructional time should focus on three critical areas:

1. Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions). Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

2. Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

3. Developing understanding of volume. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Standards for Mathematical Practice in Grade 5

The State Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that Grade 5 students complete.

Practice	Explanation and Example
1) Make sense of problems and persevere in solving them.	Mathematically proficient students in Grade 5 solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. Fifth graders may consider different representations of the problem and different solution pathways, both their own and those of other students, in order to identify and analyze correspondences among approaches. When they find that their solution pathway does not make sense, they look for another pathway that does. They check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
2) Reason abstractly and quantitatively.	Mathematically proficient students in Grade 5 recognize that a number represents a specific quantity. They extend this understanding from whole numbers to their work with fractions and decimals. This involves two processes- *decontexualizing* and *contextualizing*. Grade 5 students *decontextualize* by taking a real-world problem and writing and solving equations based on the word problem. For example, consider the task, "There are 223 of a yard of rope in the shed. If a total of 416 yard is needed for a project, how much more rope is needed?" Students decontextualize the problem by writing the equation 416−223=? and then solving it. Further, students *contextualize* the problem after they find the answer, by reasoning that 136*oooo* 112 yards of rope is the amount needed. Further, Grade 5 students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.
3) Construct viable arguments and critique the reasoning of others.	Mathematically proficient students in Grade 5 construct arguments using representations, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking through either discussions or written responses. In Grade 5, students return to their conjectures and arguments about whole numbers to determine whether they apply to fractions and decimals. For example, they might make an argument based on an area representation of multiplication to show that the distributive property applies to problems involving fractions.

Practice	Explanation and Example
4) Model with mathematics.	Mathematically proficient students in Grade 5 experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems. Example, when students encounter situations such as sharing a pan of brownies among 6 people, they might first show how to divide the brownies into 6 equal pieces using a picture of a rectangle. The rectangle divided into 6 equal pieces is a model of the essential mathematics elements of the situation. When the students write the name of each piece in relation to the whole pan as 16, they are now modeling the situation with mathematical notation.
5) Use appropriate tools strategically.	Mathematically proficient students in Grade 5 consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data. Estimation is also seen as a tool. For example, in order to solve $46-12$, a 5th grader might recognize that knowledge of equivalents of 12 is an appropriate tool: since 12 is equivalent to 36 , the result can easily be found: 16 . This practices is also related to looking for structure (SMP 7), which often results in building mathematical tools that can then be used to solve problems.
6) Attend to precision.	Mathematically proficient students in Grade 5 continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.
7) Look for and make use of structure.	Mathematically proficient students in Grade 5 look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation. For example, when 5th graders calculate 16×9, they might apply the structure of place value and the distributive property to find the product: $16\times9=(10+9)\times9=(10\times9)+(6\times9)$.
8) Look for and express regularity in repeated reasoning.	Mathematically proficient students in Grade 5 use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and

Practice	Explanation and Example
	begin to formulate generalizations. For example, 5th graders might notice a pattern in the change to the product when a factor is increased by 1: 5×7=35 $aaaaaa$ 5×8=40---the product changes by 5; 9×4=36 $aaaaaa$ 10×4=40—the product changes by 4. Fifth graders might then express this regularity by saying something like, "When you change one factor by 1, the product increases by the other factor." As students practice articulating their observations, they learn to communicate with greater precisions (SMP 6). As they explain why these generalizations must be true, they construct, critique, and compare arguments (SMP 3).

Mathematics Content Standards in Grade 5

Operations and Algebraic Thinking 5.OA

(Counting and Cardinality and Operations and Algebraic Thinking Progression K-5 Pg. 32)

Write and interpret numerical expressions.

5.OA.1. Use parentheses in numerical expressions and evaluate expressions with these symbols.

5.OA.2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "multiply the sum of 8 and 7 by 2" as* $2 \times (8 + 7)$ *because parenthetical information must be solved first. Recognize that* $3 \times (18932 + 921)$ *is three times as large as* $18932 + 921$*, without having to calculate the indicated sum or product.*

Number and Operations in Base Ten 5.NBT

(Numbers & Operations Base 10 Progression K-5 Pg. 18-20)

Understand the place value system.

5.NBT.1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.

5.NBT.2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

5.NBT.3. Read, write, and compare decimals to thousandths.

5.NBT.3a. Read and write decimals to thousandths using base-ten **numerals**, number names, expanded form, and unit form *(e.g.*

$$expanded\ form\ 47.392 = 4 \cdot 10 + 7 \cdot 1 + 3 \cdot \frac{1}{10} + 9 \cdot \frac{1}{100} + 2 \cdot \frac{1}{1000}$$

$unit\ form\ 47.392$ *= 4 tens + 7 ones + 3 tenths + 9 hundredths + 2 thousandths).*

5.NBT.3b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, <, =, and ≠relational symbols to record the results of comparisons.

5.NBT.4. Use place value understanding to round decimals to any place (Note: In fifth grade, decimals include whole numbers and decimal fractions to the hundredths place.)

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.5. Fluently (efficiently, accurately, and flexibly) multiply multi-digit whole numbers using an efficient algorithm *(ex., traditional, partial products, etc.)* based on place value understanding and the properties of operations.

5.NBT.6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the

relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
(Number and Operations Base 10 Progression K-5 Pg. 16-17)

5.NBT.7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
(Number and Operations Base 10 Progression K-5 Pg. 18-20)

Number and Operations—Fractions 5.NF

(Number and Operations – Fractions Progression Pg. 3)

Use equivalent fractions as a strategy to add and subtract fractions.
(Number and Operations – Fractions Progression Pg. 3-5)

5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,*
$\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$ *In general,* $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$

5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, *(e.g. by using visual fraction models or equations to represent the problem.)* Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. (See Table 1 to view situation types). *For example, recognize an incorrect result* $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$ *by observing that* $\frac{3}{7}<\frac{1}{2}$.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
(Number and Operations – Fractions Progression Pg. 12-14)

5.NF.3. Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b}=a\div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, *e.g. by using visual fraction models or equations to represent the problem. For example, interpret* $\frac{3}{4}$ *as the result of dividing 3 by 4, noting that* $\frac{3}{4}$ *multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size* $\frac{3}{4}$*. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

5.NF.4. Apply and extend previous understandings of multiplication (refer to 2.OA.3, 2.OA.4, 3.OA.1, 3.NF.1, 3.NF.2, 4.NF.4) to multiply a fraction or whole number by a fraction.
(Number and Operations—Fractions Progression 3–5 Pg. 12 - 13).

5.NF.4a. Interpret the product $\frac{a}{b}\cdot q$ as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a **sequence** of operations $a\cdot q\div b$. *For example, use a visual fraction model to show* $\frac{2}{3}\cdot 4=\frac{8}{3}$ *and create a story context for this equation. Do the same with* $\frac{2}{3}\cdot\frac{4}{5}=\frac{8}{15}$*. (In general,* $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$*).*

5.NF.4b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.5. Interpret multiplication as scaling (resizing), by:

5.NF.5a. Comparing the size of a product to the size of one factor based on the size of the other factor, without performing the indicated multiplication *(e.g. They see* $\left(\frac{1}{2} \cdot 3\right)$ *as half the size of 3.)*.

5.NF.5b. Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction **equivalence** $\frac{a}{b} = \frac{na}{nb}$ to the effect of multiplying $\frac{a}{b}$ by 1. *(e.g. Students may have the misconception that multiplication always produces a larger result. They need to have the conceptual understanding with examples like;* $\frac{3}{4} \times one\ dozen\ eggs$ *will have a product that is less than 12.)*

5.NF.6. Solve real world problems involving multiplication of fractions and mixed numbers, *(e.g. by using visual fraction models or equations to represent the problem)* (See Table 2 to view situation types).

5.NF.7. Apply and extend previous understandings of division (3.OA.2, 3.OA.5), to divide unit fractions by whole numbers and whole numbers by unit fractions. Division of a fraction by a fraction is not a requirement at this grade.

5.NF.7a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for* $\frac{1}{3} \div 4$*, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $\frac{1}{3} \div 4 = \frac{1}{12}$ *because* $\frac{1}{12} \cdot 4 = \frac{1}{3}$.

5.NF.7b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for* $4 \div \frac{1}{5}$*, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $4 \div \frac{1}{5} = 20$ *because* $20 \cdot \frac{1}{5} = 4$.

5.NF.7c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions*, e.g. by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share* $\frac{1}{2}$ *lb of chocolate equally? How many* $\frac{1}{3}$ *cup servings are in 2 cups of raisins?*

Measurement and Data 5.MD

Convert like measurement units within a given measurement system.

5.MD.1. Convert among different-sized standard measurement units within a given measurement system *(e.g. convert 5 cm to 0.05 m),* and use these conversions in solving multi-step, real world problems.

(Measurement and Data (measurement part) Progression K–5 Pg. 26)

Represent and interpret data.

5.MD.2. Make a data display (line plot, bar graph, pictograph) to show a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). Use operations (add, subtract, multiply) on fractions for this grade to solve problems involving information presented in the data display. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. After lunch everyone measured how much milk they had left in their containers. Make a line plot showing data to the nearest* $\frac{1}{4}$ *cup. Which value has the greatest amount? What is the total?*

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

(Measurement and Data (measurement part) Progression K–5 Pg. 26 Section 2).

5.MD.3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

- 5.MD.3a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
- 5.MD.3b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

5.MD.4. Measure volumes by counting unit cubes such as cubic cm, cubic in, cubic ft. or non-standard cubic units.

5.MD.5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

- 5.MD.5a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent three-dimensional whole-number products as volumes, *(e.g. to represent the associative property of multiplication.)*
- 5.MD.5b. Apply the formulas $V = l \cdot w \cdot h$ *and* $V = B \cdot h$ (*B represents the area of the base)* for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
- 5.MD.5c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Students find the volume of each rectangular prism that is decomposed from the original figure and add the individual volumes to find the total volume.

Geometry 5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.

(Geometry Progression K-6 Pg. 17 and graphic from Pg. 17-18)

5.G.1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond *(e.g. x-axis and x-coordinate, y-axis and y-coordinate)*.

5.G.2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. *(e.g. plotting the relationship between two positive quantities such as maps, coordinate grid games (such as Battleship), time/temperature, time/distance, cost/quantity, etc.).*

Classify two-dimensional figures into categories based on their properties

(Geometry Progression K-6 Pg. 17 and graphic from Pg. 18)

5.G.3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

5.G.4. Classify two-dimensional figures in a hierarchy based on properties.